Basic Antenna Relationships and Design Considerations for Rectennas

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Antenna Equations

In a system comprising a transmitter producing a power output of P_t at a frequency whose wavelength is λ, a transmitting antenna with a numerical gain of G_t and a receiving antenna with a numerical gain of G_r at a distance of R meters from the transmitter, the Friis Transmission Equation states that the power at the output of the receiving antenna is:

$$P_r = P_t G_t G_r (\lambda/4\pi R)^2 \qquad (1)$$

In the above, the wavelength of the signal (λ) is defined to be the speed of light (C = 3×10^8 meters/second) divided by the frequency:

$$\lambda = C/f \qquad (2)$$

At 2.45 GHz, λ is 0.1224 meters

Note that as R becomes small in the Friis equation, the equation predicts that P_r increases without limit and that P_r becomes greater than P_t when $G_t G_r (\lambda/4\pi R)^2$ becomes greater than unity. Since this is plainly incorrect (an antenna cannot create energy), the formula is only useable when $R > (G_t G_r)^{1/2} \lambda/4\pi$ and is quite inaccurate even at this distance. This is one of many relationships that demonstrate near-field effects where formulas such as the Friis equation become inaccurate. (Note: A more correct point of view is that the antenna gains used in the Friis equation are only valid at a great distance from the antenna. At close distances the effective gains are reduced.) The formula becomes relatively accurate beyond the Raleigh Distance defined as $R = 2D^2/\lambda$ where D is the largest dimension of the antenna. As an example, at 2.45 GHz (λ = .1224 meter) with 14 dB (25x) gain antennas that have an active radiating area about 7 inches (.180 meters) square and largest (diagonal) dimension of 10 inches (0.254 meters), the Raleigh distance for each antenna is R = 1.054 meters (41.5 inches). (This example antenna is a HyperLink Technologies Model RE14P-RSP. See http://www.hyperlinktech.com/web/re14p.php.) Since each antenna is subject to the Raleigh distance, the antennas should be spaced at twice this distance (2.108 meters) to approach far-field performance. This does not mean the antennas will not work at closer distance, only that the Friis equation using the stated antenna gains becomes inaccurate. At twice the Raleigh Distance (R = 2.108 meters) in this example with a transmit power (P_t) of 1 watt and antenna gains of 25, the received power is predicted to be:

$P_r = P_tG_tG_r (\lambda/4\pi R)^2$

$P_r = 1 \times 25 \times 25 * (.1224/(4 \times 3.14 \times 2.108))^2$

$P_r = 0.0133$ watts (1.33% coupling efficiency)

Note that this in not a particularly accurate estimate since the Raleigh distance is merely the point at which far field relationships become larger that near field relationships. This estimate might be expected to have an error as large as 2x (generally predicting higher than actual measured results) since near and far field effects are approximately equal. As the antenna spacing is decreased below 2.108 meters, the received power will continue to increase but not as rapidly as the Friis equation predicts. There will also be seen a cyclic variation in received power with distance that becomes increasingly noticeable as the distance decreases. This occurs because the two antennas are coupling directly to one another instead of into free space and the impedance matching and energy distribution of the antennas is no longer correct except at discrete distances. This is not particularly harmful except that the optimum spacing must be found by experiment once all the system components are in hand.

An isotropic (point source) radiator radiates equally in all directions which leads to the conclusion that at any distance R from the source, the transmitted power is spread uniformly over the surface of a sphere of that radius. The surface area of a sphere is $A = 4\pi R^2$ and so the power density is $P_{dens} = P_t/4\pi R^2$. An antenna produces gain by concentrating the radiation into a narrow beam and in this case the power density in the direction of maximum radiation is increased by the gain of the antenna so that:

$$P_{dens} = P_tG_t/4\pi R^2 \qquad (3)$$

which expresses the power density (in watts/meter^2) at a distance of R meters from the transmitting antenna. (Note: Power density is often expressed in milliwatts/cm^2. Because there are 1000 milliwatts per watt and 10,000 cm^2 per meter^2, the power density in milliwatts/cm^2 is 1/10 the power density in watts/meter^2.) This is one of the basic relationships used to derive the Friis equation.

Equation 3 is valid at great distance from the antenna but is very inaccurate at close distance. The author sometimes makes near field predictions by assuming that the transmitted signal emanates from a point behind the antenna face using the following method:

1. Assume that the power density begins at the face of the antenna with the power spread uniformly over the antenna area such that:

$$P_{dens} = P_t/A$$

Substitute this into equation (3) to calculate an apparent point of origin ($R = R_A$) for the transmit signal:

$$P_t/A = P_tG_t/4\pi R_A^2$$

$$R_A^2 = G_tA/4\pi$$

$$R_A = (G_tA/4\pi)^{1/2}$$

2. If beam patterns are available for the antenna, calculate two additional apparent origins from the horizontal and vertical beam widths (θ_H and θ_V) and antenna height (H) and width (W) :

$$R_{AH} = ½W/\tan(\theta_H/2)$$

$$R_{AV} = ½H/\tan(\theta_V/2)$$

Take the largest of the above three R_A values as the effective R_A in the following. This radius is the "apparent" source of the transmitted signal as if it was from a source of zero dimensions (point source) and should be added to the actual distance from the antenna to predict field strength. Then the Friis equation may be modified as:

$$P_r = P_tG_tG_r\ (\lambda/4\pi(R + R_A)^2$$

And equation 3 may be modified as:

$$P_{dens} = P_tG_t/4\pi(R + R_A)^2$$

The predictions of the above two equations are not particularly accurate because of interaction between the antennas and measurements usually run about 70 to 75 percent of the predicted values but this may be used for first-order "guesstimates". For the example antennas mentioned above, effective R_A is about 13.1 inches determined from the beam pattern equations and the predicted received power and power density for a transmit power of one watt at a number of distances from the antenna is as follows:

Distance (Inches)	Received Power predicted by equation (watts)	Predicted Received Power reduced by 22.5% (watts)	Power Density predicted by equation (mw/cm^2)
4	0.32	0.23	1.06
8	0.21	0.15	0.70
12	0.15	0.11	0.49
16	0.11	0.08	0.37

If a receiving antenna of cross-sectional area (called the physical aperture) of A square meters is placed in the field radiated by the antenna described by equation 3, it will intercept a total power of $P_{max} = A * P_{dens}$ watts. This is an upper limit on the amount of power the receiving antenna can produce. Except for a few special cases where the antenna is very small, real world antennas always produce less than this upper limit and the ratio of the actual received power to the maximum is called the aperture efficiency of the antenna:

$$E_A = P_r/P_{max}$$

Comparing equation 3 with the Friis equation suggests that the Friis equation may be split into two parts, where equation 3 gives the power density produced by the transmitting system and the following gives the power delivered at the output of a receiving antenna as a result of that power density:

$$P_r = P_tG_tG_r\,(\lambda/4\pi R)^2$$

$$P_r = (P_tG_t/4\pi R^2) * (G_r\lambda^2/4\pi)$$

$$P_r = P_{dens} * (Gr\lambda^2/4\pi) \tag{4}$$

$G_r\lambda^2/4\pi$ is called the effective aperture A_e:

$$A_e = G_r\lambda^2/4\pi \tag{5}$$

and

$$P_r = P_{dens} * A_e$$

In an ideal antenna, the effective aperture will be the same as the physical aperture. Small antennas can have an effective aperture greater than the physical area of the antenna because the antenna "sucks in" energy from nearby space, but large antennas will always have an effective aperture less than the physical area. Now from above the aperture efficiency may be expressed as:

$$E_A = P_r/P_{max}$$

$$E_A = (P_{dens} * A_e)/(A * P_{dens})$$

$$E_A = A_e/A$$

which is merely the ratio of the effective aperture to the physical aperture.

In all antennas where beam shape is important, the energy distribution across the face of the antenna must be tapered to a small value at the edges of the antenna to minimize sidelobe radiation. (A Gaussian taper is generally taken to be the optimal taper function.)

This causes a reduction in the gain and aperture efficiency with the general result that antennas with good beam shape will not have an aperture efficiency much better than 50%. This means that the antenna physical area must be made larger to obtain the required gain but is otherwise not harmful. Gain and aperture efficiency are further reduced by any losses in the feed structure or radiating element. These losses represent lost power that cannot be recovered.

Since $E_A = A_e/A$, combining with equation 5 gives:

$$E_A = G_r\lambda^2/(4\pi A) \quad (6)$$

This may be rearranged to predict the gain of an antenna as:

$$G_r = 4\pi E_A A/\lambda^2 \quad (7)$$

Since the antenna area will be the product of two linear dimensions such as height x width for a rectangle or πr^2 for a circle, equation 7 can also be stated as:

$$G_r = 4\pi E_A N \quad (8)$$

where the antenna area N is in units of wavelengths squared (A/λ^2).

For the example antenna above where the antenna active area is about 7 inches x 7 inches (0.180 x 0.180 meters or 1.47 x 1.47 wavelengths) and the gain is 25x, from equation 6 the aperture efficiency is about 0.92. (Remarkably good for an antenna of this size in which aperture efficiency was probably not the primary goal. This leads one to suspect the manufacturer may be overly optimistic about the rated gain.)

Signal strength is often expressed as the equivalent electric field strength. Power density in watts/meter^2 may be converted into field strength in volts/meter by

$$E = (120\pi P_{dens})^{1/2} \quad (9)$$

or,

$$P_{dens} = E^2/120\pi \quad (10)$$

Substituting the equation for P_{dens} into the equation for electric field strength gives:

$$E = (30P_tG_t)^{1/2}/R, \quad (11)$$

which describes the electric field strength at a distance of R meters produced by a transmitter with a power output of P_t watts and a transmitting antenna with a gain of G_t.

Antenna Design Considerations

There exist a large variety of antennas, each having certain advantages and disadvantages. Commonly used types include the horn, parabolic reflector, dipole, patch, Yagi and collinear arrays of the above. In some applications the antenna is required to radiate power equally in all directions while in others the goal is to concentrate the signal in a single direction. Power beaming falls into this latter category since the only useful radiation from the antenna is what arrives at the receiving antenna. (Note: Antennas behave the same way whether transmitting or receiving. It is usually easier to describe antenna behavior in the context of transmitting and that point of view will be used in this discussion, but all characteristics such as beamwidth, gain and aperture efficiency apply equally to either case.) Gain is accomplished by spreading the antenna radiation over a large antenna area and insuring that the radiation from all parts of the area is launched with equal phase in the direction of radiation. This is not difficult for small antennas but imposes severe demands on dimensional accuracy and stability for large antennas.

Horn antennas require considerable depth along the axis of radiation. The depth can be reduced somewhat by providing a focussing lens, but in either case the horn quickly becomes large and heavy. In addition, the power-handling capability is limited by the fact that all of the power is concentrated at the throat of the horn leading to problems with voltage breakdown (arcing) and excessive heating of the walls. The effective aperture of horn antennas is typically limited to 50 to 60 percent because the energy density at the face of the horn tapers to zero at the walls parallel to the electric field.

Parabolic reflectors are fed by a radiating element (feed) that launches energy towards a parabolic reflecting surface from the focal point of the reflector. The parabolic shape of the reflector causes all of this energy to be reflected in the direction of radiation with equal phase. This requires high dimensional accuracy (to a fraction of a wavelength) of the reflecting surface and also suffers from the same power concentration problem as the horn because all of the transmitted power is concentrated in the radiating element at the feed point (commonly a small horn). The effective aperture of a parabolic reflector also tends to be limited to 50 to 60 percent because the radiating element either does not illuminate the entire reflector area or else part of the energy radiated from the feed spills over the edges of the reflector.

Dipoles are among the simplest and most basic of antennas. A dipole is a ½ wavelength long wire split and fed at the center. Dipoles produce strongest radiation in all directions perpendicular to the wire, decreasing to zero in a direction parallel to the wire. The radiation can be concentrated in one direction by placing the dipole 1/4 wavelength above a conducting surface ("ground plane"). The dipole is an example of the case where the effective aperture is greater than the physical area. If the dipole is made of a thin wire, the physical area is quite small but the signal intercepted by the dipole remains relatively constant independent of the wire size. The gain of a dipole in free space is 2.15 dBi (1.64x) over an isotropic radiator (Gain over an isotropic radiator is abbreviated dBi). When placed ¼ wavelength above a conducting surface (ground plane) the gain increases to about 6 dBi (4x) in a direction perpendicular to the ground plane.

A patch antenna is a thin rectangle of conducting material suspended a small fraction of a wavelength above and parallel to a ground plane. The dimensions of the rectangle determine the frequency of the antenna. Radiation is perpendicular to the plane of the rectangle and ground plane and the radiation pattern is similar to the dipole spaced ¼ wavelength above a ground plane. A dielectric material (such as circuit board material) is usually used to support the patch above the ground plane. Performance of a patch antenna is not quite as good as a dipole. Gain is usually about 5 dBi and losses are somewhat higher than a dipole. The loss properties of the dielectric material are important to obtain good efficiency. In spite of this, patch antennas are frequently used because they may be constructed economically.

Yagi antennas add parasitic elements (a reflector and one or more directors) to a dipole to concentrate radiation in one direction. Energy is fed only to the dipole and the other elements interact with the radiated field from the dipole. Element-to-element spacing is usually about 0.2 wavelengths so a 4 element Yagi (dipole plus reflector plus two directors) is 0.6 wavelengths long along the axis of maximum radiation and about 0.5 wavelengths perpendicular to the axis of maximum radiation. A 4 element Yagi can produce a gain of about 9 dBi. A Yagi can be suspended over a ground plane pointed away from the ground plane with little effect on performance.

Both dipoles and Yagis present problems with the supporting structure because they must be suspended in space above a ground plane. This leads to an antenna that is relatively "thick" in the direction of radiation and somewhat fragile in comparison to a patch antenna.

Any of the above basic antennas can be arranged into collinear arrays, in which transmit energy is fed to all of the individual antennas. The individual elements are usually spaced about ½ wavelength apart to obtain the best compromise of gain versus number of elements. The gain ideally increases in direct proportion to the number of antenna elements but in the real world will be somewhat less than this due to interaction of the individual elements. This interaction increases as the element spacing is decreased. Aperture efficiency can be increased at the expense of beam shape and number of elements by close spacing of the individual elements and by not tapering the energy at the edges of the array. Collinear arrays are not practical for large antennas because of losses in the transmission lines feeding the elements but provide good performance for arrays of a few wavelengths overall dimensions.

The requirements for the transmitting antenna are quite different than the requirements for the receiving antenna in a power beaming application. The transmitting antenna must focus the radiated energy into a narrow beam with minimal sidelobes while the receiving antenna must efficiently recover the energy incident on the antenna. To accomplish focusing, the transmitted energy must be distributed over a large area with accurate control of the amplitude and phase of the signal over that area. As the antenna is made larger, the point is soon reached where feeding the power across this area introduces unacceptable losses. In addition, maintaining equal phase across the face of the array imposes severe requirements on the mechanical accuracy and stability of the structure. Fortunately modern technology provides solutions to both of these problems. Instead of generating microwave power in a high power transmitter and then distributing to a large number of radiating elements, power may instead be generated at relatively low levels (perhaps 100 watts per transmitter) and the output of each transmitter may be distributed to only a small number of nearby radiating elements. This avoids the high distribution losses inherent with higher power transmitters and a large number of elements. The remaining problem is that all of the transmitters in a large array must be synchronized in phase. This problem can be solved by transmitting a low power pilot signal from the receiving point to the transmitting array. Each transmitter receives this pilot signal and uses the signal to adjust the phase of its transmitted signal to produce the required result. This greatly reduces the required accuracy since each transmitter can automatically compensate for errors in its location within the array.

For the receiving antenna, not only is there no need to focus the antenna into a narrow beam, but in fact the opposite is of advantage since it can eliminate the need to accurately aim the receiving antenna at the transmitter. Thus, gain and beam shape are not of primary importance. Conversely, high aperture efficiency is the primary goal since this is a measure of how much of the energy incident on the antenna is converted into received power. Any of the basic antennas described above, or small collinear arrays of these antennas are suitable solutions to this problem. While not providing quite as good performance of some of the other solutions, patch antennas are attractive because they can be fabricated inexpensively using printed circuit board techniques and the rectifying element can also be mounted on this circuit board. Any required number of these basic modules can be spread over a large area and the DC output from the modules can be transmitted directly or converted to low frequency AC and transmitted to a central collection point without high losses.